Program

Tuesday

14:30-15:20 15:20-15:45	Welcoming Olga Mula Para CEIM or the incredible journey of doing parareal and CEIM with Yven Maday
	Para-GEIM or the incredible journey of doing parareal and GEIM with Yvon Maday
15:45-16:15	Coffee break 🖢
16:15-17:05	Gero Friesecke X-rays and the determination of atomic structure – a new twist?
17:05-17:30	Gianluigi Rozza Reduced Order Methods for parametric Fluid-Structure Interaction problems: applications to haemodynamics
17:30-17:55	Marco Verani On Adaptive Spectral Galerkin Methods with Dynamic Marking
19:30	Dinner
Wednesday	
09:00-09:50	Mark Ainsworth Fractional Cahn-Hilliard Equation(s): Analysis, Properties and Approximation
09:50-10:15	Peng Chen Hessian-based sampling for model order reduction
10:15-10:45	Coffee break 🖤
10:45-11:35	Paul Fischer Spectral Elements for Large Turbulent Flow Simulations
11:35-12:00	Ulrik Skre Fjordholm Statistical solutions of hyperbolic conservation laws
12:00-14:30	Lunch break
14:30-15:20	Jan Hesthaven Advances in reduced order modeling
15:20-15:45	Clair Poignard Mathematical electroporation modeling, from cell scale towards clinical applications
15:45-16:15	Coffee break 🖤
16:15-17:05	Ronald DeVore Data Fitting
17:05-17:30	Edwige Godlewski An adapted model reduction method for complex biological systems
17:30-17:55	Frédéric Coquel CEMRACS: a success story
19:30	Banquet

Thursday

09:00-09:50	
09:50-10:15	Multigrid Interpretations of the Parareal Algorithm Virginie Ehrlacher
	An embedded corrector problem for stochastic homogenization
10:15-10:45	Coffee break 🖐
10:45-11:35	Bertrand Maury
11:35-12:00	Modelling of the respiratory system: the notion of airway resistance Ludovic Chamoin
	Recent advances in a posteriori error estimation and adaptivity for PGD reduced models
12:00-14:30	Lunch break
14:30-15:20	Jie Shen
15:20-15:45	Efficient and accurate spectral methods for PDEs with singular solutions Nicole Poussineau
13.20-13.43	From oil to glass, via a PhD with Yvon Maday
15:45-16:15	Coffee break 🛎
16:15-17:05	Valérie Perrier
17:05-17:30	Adaptive wavelet methods for PDEs : the story, and some recent advances Andrea Manzoni
	Reduced basis methods for parametrized PDEs: how to enhance efficiency
17:30-17:55	and accuracy in complex problems Matthieu Hillairet
	Regularity issues in fluid-structure interaction problems
19:30	Dinner
Friday	
09:00-09:50	Claude Le Bris
09:50-10:15	A selection of topics in numerical homogenization Antoine Levitt
05.50 10.15	Numerical analysis of Brillouin zone integration methods
10:15-10:45	Coffee break 🛎
10:45-11:35	Saloua Mani-Aouadi Towards the modelling the Purkinje/ myocardium coupled problem: A well
	posedness analysis
11:35-12:00	Posters
12:00	Closing

Fractional Cahn-Hilliard Equation(s): Analysis, Properties and Approximation

Mark Ainsworth

Brown University, USA

The classical Cahn-Hilliard equation [1] is a non-linear, fourth order in space, parabolic partial differential equation which is often used as a diffuse interface model for the phase separation of a binary alloy. Despite the widespread adoption of the model, there are good reasons for preferring models in which fractional spatial derivatives appear [2,3]. We consider two such Fractional Cahn-Hilliard equations (FCHE). The first [4] corresponds to considering a gradient flow of the free energy functional in a negative order Sobolev space H^{α} , $\alpha \in [0,1]$ where the choice $\alpha = 1$ corresponds to the classical Cahn-Hilliard equation whilst the choice $\alpha = 0$ recovers the Allen-Cahn equation. It is shown that the equation preserves mass for all positive values of fractional order α and that it indeed reduces the free energy. The well-posedness of the problem is established in the sense that the H^1 -norm of the solution remains uniformly bounded. We then turn to the delicate question of the L_{∞} boundedness of the solution and establish an L_{∞} bound for the FCHE in the case where the non-linearity is a quartic polynomial. As a consequence of the estimates, we are able to show that the Fourier-Galerkin method delivers a spectral rate of convergence for the FCHE in the case of a semi-discrete approximation scheme. Finally, we present results obtained using computational simulation of the FCHE for a variety of choices of fractional order α . We then consider an alternative FCHE [3,5] in which the free energy functional involves a fractional order derivative. This is a joint work with Zhiping Mao.

[1] J.W. Cahn and J.E. Hilliard, *Free energy of a non-uniform system*. *I. Interfacial Free Energy*, J. Chem. Phys, 28, 258–267 (1958)

[2] L. Caffarelli and E. Valdinoci, *A Priori Bounds for solutions of non-local evoluation PDE*, Springer, Milan 2013.

[3] G. Palatucci and O. Savin, *Local and global minimisers for a variational energy involving a fractional norm*, Ann. Mat. Pura Appl., 4, 673–718 (2014).

[4] M. Ainsworth and Z. Mao, *Analysis and Approximation of a Fractional Cahn-Hilliard Equation*, (In review, 2016).

[5] M. Ainsworth and Z. Mao, *Well-posedness of the Cahn-Hilliard Equation with Fractional Free Energy and Its Fourier-Galerkin Discretization*, (In review, 2017).

Recent advances in a posteriori error estimation and adaptivity for PGD Ludovic Chamoin

LMT, ENS Paris-Saclay

Due to the increasing number of high-dimensional approximation problems, which naturally arise in many situations such as stochastic analysis and uncertainty quantification, model reduction techniques have been the object of a growing interest in research and industry. Tensor methods are among the most prominent tools for the numerical solution of such problems; in many practical applications, the approximation of high-dimensional solutions is made computationally tractable by using low-rank tensor formats. In particular, an appealing technique based on low-rank canonical format and referred to as Proper Generalized Decomposition (PGD) was introduced and successfully used in many applications of Computational Mechanics. Contrary to POD, the PGD approximation does not require any knowledge on the solution, and operates in an iterative strategy in which basis functions (or modes) are computed on the fly, by solving eigenvalue problems. Even though the PGD is very effective for a large class of high-dimensional problems, it is crucial to derive verification tools in order to both certify the quality of the approximate PGD solution and drive efficient numerical algorithms. Among the few a posteriori error estimation tools available in the PGD framework, an approach based on duality analysis and the concept of constitutive relation error (CRE) was developed during the last years for linear elliptic or parabolic problems. It provides for strict bounds on both global error and error on outputs of interest. It also enables to assess contributions of various error sources (space/time discretizations, truncation of the PGD decomposition, ...), which helps driving adaptive strategies and saving CPU time and memory space. In the present work, we address recent advances performed in this PGD-verification method. We consider problems with parameters of all kinds (material properties, geometry,...), investigate the case of PGD with separation of space coordinates, and extend the verification framework to nonlinear problems solved with the LATIN-PGD method. This is a joint work with P. Ladevèze and P.E. Allier, LMT, ENS Paris-Saclay.

Hessian-based sampling for model order reduction Peng Chen Cornell University

How to choose the training samples for the construction of reduced order model in high-dimensional parameter space is an interesting and important question. Different from the commonly used sampling methods such as random sampling, adaptive sampling, sparse grid sampling, etc., we propose a new sampling algorithm based on the Hessian information of a given quantity of interest. This sampling method is demonstrated to be more efficient than random sampling for goal-oriented model order reduction.

CEMRACS: a success story Frédéric Coquel CNRS, Ecole Polytechnique

Data Fitting Ronald DeVore Texas A&M University

A common scientific problem is that we are given some data about a function f and we wish to use this information to either (i) approximate f or (ii) answer some question about f called a quantity of interest. We discuss recent results on data fitting inspired by the work of Yvon Maday which determine optimal algorithms for the two scenarios above under the assumption that f is in a model class described by approximation.

An embedded corrector problem for stochastic homogenization Virginie Ehrlacher

CERMICS – ENPC

A very efficient algorithm has recently been introduced in [1] in order to approximate the solution of implicit solvation models for molecules. The main ingredient of this algorithm relies in the clever use of a boundary integral formulation of the problem to solve. The aim of this talk is to present how such an algorithm can be adapted in order to compute efficiently effective coefficients in stochastic homogenization for random media with spherical inclusions. To this aim, the definition of new approximate corrector problems and approximate effective coefficients is needed and convergence results in the spirit of [2] are proved for this new formulation. Some numerical test cases will illustrate the behaviour of this method.

This is a joint work with Eric Cancès, Frédéric Legoll and Benjamin Stamm

[1] "Domain decomposition for implicit solvation models", Eric Cancès, Yvon Maday, Benjamin Stamm, The Journal of Chemical Physics 139 (2013) 054111

[2] "Approximations of effective coefficients in stochastic homogenization", Alain Bourgeat, Andrey Piatnitski, Annales de l'institut Henri Poincaré (B) Probabilités et Statistiques 40 (2004) page 153-165

Spectral Elements for Large Turbulent Flow Simulations Paul Fischer

University of Illinois Urbana-Champaign, Argonne National Laboratory The introduction of the spectral element method (SEM) in the 1980s [1,2] set the stage for practical and efficient simulation of turbulence in complex domains. The exponential convergence of the underlying discretization yielded low numerical dissipation and dispersion, and the minimal (C^0) continuity requirement provided a natural framework for efficient distributed-memory parallel computing in which operator evaluation could be evaluated with a communication-minimal unit-depth stencil. Importantly, the local structure of the SEM, coupled with Lagrangian bases on high-order Gauss-Lobatto-Legendre quadrature nodes, allowed for fast tensor-product-based operator evaluation, with per-gridpoint memory access requirements that are equivalent to standard finite-difference stencils and that are substantially lower than standard finite-element methods. While the theory for the SEM was rigorously established in the early work of Bernardi, Maday, Patera and contemporaries, several additional ingredients were required to make it practicable for high Reynolds number flows. Among these, we mention stabilization and dealiasing [3,4], stable high-order timesteppers [5], and scalable solvers [6,7,8].

In this talk, we give a brief overview of these historical developments and then discuss recent developments in very large scale SEM simulations of turbulence—involving millions of elements, billions of gridpoints, and hundreds of thousands of processors—for a variety of applications in science and industry. We illustrate how features intrinsic to the SEM yield significant reductions in overall computational costs and we close with some remarks concerning prospects for turbulence simulation in the forthcoming exascale era.

[1] A.T. Patera. A spectral element method for fluid dynamics : laminar flow in a channel expansion. J. Comput. Phys., 54:468–488, 1984.

[2] Y. Maday and A.T. Patera. Spectral element methods for the Navier-Stokes equations. In A.K. Noor and J.T. Oden, editors, State-of-the-Art Surveys in Computational Mechanics, pages 71–143. ASME, New York, 1989.

[3] P.F. Fischer and J.S. Mullen. Filter-based stabilization of spectral element methods. Comptes rendus de l'Académie des sciences, Série I- Analyse numérique, 332:265–270, 2001.

[4] J. Malm, P. Schlatter, P.F. Fischer, and D.S. Henningson. Stabilization of the spectral-element method in convection dominated flows by recovery of skew symmetry. J. Sci. Comp., 57:254–277, 2013.

[5] Y. Maday, A.T. Patera, and E.M. Rønquist. An operator-integration-factor splitting method for time-dependent problems: Application to incompressible fluid flow. J. Sci. Comput., 5:263–292, 1990.

[6] J. W. Lottes and P. F. Fischer. Hybrid multigrid/Schwarz algorithms for the spectral element method. J. Sci. Comput., 24:45–78, 2005.

[7] J.W. Lottes. Independent quality measures for symmetric AMG components. Preprint ANL/MCS-P1820-0111, 2011.

[8] H.M. Tufo and P.F. Fischer. Fast parallel direct solvers for coarse-grid problems. J. Parallel Distrib. Comput., 61:151–177, 2001.

Statistical solutions of hyperbolic conservation laws Ulrik Skre Fjordholm

Department of Mathematical Sciences, NTNU

For multi-D hyperbolic conservation laws such as the (in)compressible Euler equations, there is a great lack of stability, existence and uniqueness theory, and certain Cauchy problems are provably ill-posed. Numerical methods for multi-D gas flow can be highly unstable with respect to initial data and show no sign of convergence. These facts indicate an inherent uncertainty in the solution, even when the initial data is given exactly. We build the concept of statistical solutions, which are probability measures on L^1 satisfying certain evolution equations. For scalar conservation laws we impose entropy conditions which imply existence, uniqueness and stability.

X-rays and the determination of atomic structure – a new twist? Gero Friesecke

Technische Universität München

My talk will explore - at the level of modelling and simulation - the possiblity of novel X-ray methods for the determination of the atomic structure of highly regular but not periodic molecules. The details were worked out for helical structures. These include carbon nanotubes, the necks and tails of many viruses, and some common proteins (actin, collagen). The quest for novel methods is motivated by the fact that current X-ray methods, while hugely successful, have important shortcomings. A native helical assembly of proteins either has to be broken at the outset and the proteins crystallized, which is difficult and may lead to non-native forms; or one confines oneself to imaging of noncrystalline samples (fiber diffraction, CDI. cryo-EM, ...), which is less invasive but has lower resolution. As regards helical structures, we worked out exact solutions to Maxwell's equations ('twisted X-rays') which yield discrete Bragg-type diffraction patterns, in the same way in which plane waves used in current X-ray crystallography yield discrete patterns for crystals. We demonstrated the possiblity of structure determination by recovering the structure of the Pf1 virus (Protein data bank entry 1pfi) from its simulated diffraction data under twisted X-rays. An interesting feature of the required waves is that they carry orbital angular momentum (OAM); unfortunately OAM waves can at present only be realized experimentally down to the regime of soft-Xray wavelengths, not hard-Xray wavelengths needed for atomic resolution. Joint work with Dominik Juestel (TUM) and Richard James (University of Minnesota), SIAM J. Appl. Math. 76 (3), 2016, and Acta Cryst. A72, 190, 2016.

Multigrid Interpretations of the Parareal Algorithm Martin Gander

Université de Genève

The original parareal algorithm is a two level method. If one wants to extend it to a multilevel method, it is instructive to first interpret the parareal algorithm as a two grid method, and to identify the corresponding multigrid components like the smoother, the restriction and prolongation and the coarse grid operator. Different such interpretations are possible, and will be explained. One of these interpretations leads to MGRIT, where the F-smoother used in parareal is replaced by an FCF-smoother. This can in turn be interpreted as an overlapping variant of the parareal algorithm, and reveals that overlap might be beneficial in some situations.

An adapted model reduction method for complex biological systems Edwige Godlewski

Lab. Jacques-Louis Lions - UPMC

We are interested to a model reduction method which allows to simulate some particular features of complex phenomena in biology. We consider a well known biological system for which we develop a modelisation strategy together with suitable tools in order to describe the main properties of the system.

Advances in reduced order modeling Jan Hesthaven

Chair of Computational Mathematics and Simulation Science, EPFL, Switzerland The development of reduced order models for complex applications, offering the promise for rapid and accurate evaluation of the output of complex models under parameterized variation, remains a very active research area. However, many challenges remain to secure the flexibility, robustness, and efficiency needed for general large scale applications, in particular for nonlinear and/or time-dependent problems.

In this talk, we discuss developments in two different directions. In the first part, we discuss recent developments of reduced methods that conserve chosen invariants for nonlinear time-dependent problems. We pay particular attention to the development of reduced models for Hamiltonian problems and propose a greedy approach to build the basis. The performance of the approach is demonstrated for both ODEs and PDEs and is extended to more general dissipative problems through the notion of port-Hamiltonians.

The second part of the talk discusses the combination of reduced order modeling for nonlinear problems with the use of neural networks to overcome known problems of on-line efficiency for general nonlinear problems. We discuss the general idea in which training of the neural network becomes part of the offline part and demonstrate its potential through a number of examples. This is a joint work with B.F. Afkram, N. Ripamonti and S. Ubbiali.

Regularity issues in fluid-structure interaction problems Matthieu Hillairet

Université de Montpellier

In order to construct an existence theory for system of pdes (or pde/odes) modeling fluid-structure problems, a classical difficulty is that the fluid domain may become singular. For instance, in the case of rigid bodies moving in a fluid, contacts may occur between the bodies, or, in the case of a flow inside an elastic channel, the shape of the channel may not remain smooth or collapse. In this talk, I will focus on these two examples and discuss the possible loss of regularity of the fluid domain in presence of an incompressible viscous fluid.

A selection of topics in numerical homogenization Claude Le Bris

Ecole des Ponts and Inria

We will review some recent works in numerical homogenization and multiscale science. The numerical approaches presented all aim at addressing problems where no periodicity of the microstructure is assumed. The questions include: approximation of the homogenized coefficients in the presence of incomplete information, computation of the numerical solution of problems on heterogeneous perforated domains, issues related with random parameters, etc. The works are joint works with Frédéric Legoll and various other collaborators.

Numerical analysis of Brillouin zone integration methods

Antoine Levitt

CERMICS - ENPC

Spectral properties of Schrödinger operators with periodic potentials can be computed via the Bloch-Floquet theory. This expresses quantities of interest as integrals over the Brillouin zone, a d-dimensional torus, of spectral properties of a parametrized operator acting on a finite domain, yielding efficient computational schemes. I will present a numerical analysis of the methods used to compute these integrals in condensed matter physics. This is joint work with E. Cancès, V. Ehrlacher, D. Gontier and D. Lombardi.

Towards the modelling the Purkinje/ myocardium coupled problem: A well Saloua Mani-Aouadi

University Tunis El Manar, Faculty of Sciences of Tunis, Tunisia

The Purkinje network is the specialized conduction system in the heart. It is coupled to the myocardium and ensures the physiological spread of the electrical wave in the ventricles. Inspired by [1], we perform the coupling using a monodomain/bidomain model where both the myocardium and the intra-myocardium branch of the Purkinje's network are treated with a bidomain model and the extra-myocardium branch is treated with a monodamain model. The three-dimensional equations for a fiber are reduced to one dimensional equations. The resulting problem is an 1D/3D coupled reaction-diffusion system. We discretize in time this system by a semi-implicit scheme. We then write a variational formulation in a non standard functional frame with specific weighted Sobolev spaces, as introduced in [2]. We prove the existence and uniqueness of solution for the Purkinje/myocardium coupled problem. We discretize in space by the finite element P1-Lagrange and do some numerical tests.

Secondly, we reconsider the coupling in a fully monodomain framework. We present different strategies based on different time discretization of the coupling terms. We prove the stability of the considered schemes under the same restriction of the time step. Moreover, we show that the energy is slightly affected by the time splitting schemes. We conduct 3D and 2D simulations which confirm our predictions.

This is a joint work with W. Mbarki, from the University Tunis El Manar, Faculty of Sciences of Tunis, Tunisia, mbarki-wajih@live.fr, and N. Zemzemi, INRIA-Bordeaux, Sud-Ouest, Carmen project, France, nejib.zemzemi@inria.fr.

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Reduced basis methods for parametrized PDEs: how to enhance efficiency Andrea Manzoni

EPFL, Lausanne

Reduced basis (RB) methods represent a reliable and efficient approach for the numerical approximation of problems involving the repeated solution of differential equations arising from engineering and applied sciences. Noteworthy examples include partial differential equations (PDEs) depending on several parameters, PDE-constrained optimization, data assimilation and uncertainty quantification problems. When dealing with more complex nonaffine and/or nonlinear problems, several challenges have to be faced to ensure accuracy and computational efficiency.

These involve, among others, the need of (i) generating the reduced problem in non-intrusive and purely algebraic way; (ii) estimating the reduction errors or providing effective error surrogates; (iii) approximating manifolds of large intrinsic dimension with low-dimensional subspaces through possibly nonlinear or localized model order reduction algorithms. In this talk I will show how to combine some recent reduc! tion and hyper-reduction techniques to solve a variety of computationally-intensive problems ranging, e.g., from fluid dynamics on domains with varying shape to cardiac electrophysiology and parameter estimation in a Bayesian setting.

Modelling of the respiratory system: the notion of airway resistance Bertrand Maury

Université Paris-Sud & DMA, Ecole Normale Supérieure de Paris

About 15 years ago, Yvon Maday proposed to some colleagues in LJLL to start a project on the respiratory system. None of us had a slightest clue on this domain, so we enthusiastically accepted the proposition. It started a very exciting and rewarding long lasting story which is still going on today. I would like to present some aspects of this story, from a particular standpoint: the so-called Airway Resistance (Raw), which links the pressure drop accross the system to the air flux, is commonly measured in clinical practice, and it is also a crucial ingredient in the modeling of the ventilation process. Yet, beyond the straight analogy with Ohm's law for electric wires or network, the proper definition of this notion raises deep issues in terms of modelling, mathematical setting, and actual numerical computations.

This work has been initiated in collaboration with Y. Maday and C. Grandmont, and continued with various co-workers, in particular A. Decoene, H. Guénard, and S. Martin.

Para-GEIM or the incredible journey of doing parareal and GEIM with Yvon Maday Olga Mula

Paris Dauphine University

In this talk, I will present two recent contributions that I have made in collaboration with Yvon Maday. The first one is related to the improvement of the parallel efficiency of the parareal in time algorithm when one uses a degraded fine solver. The second one is related to the field of reduced basis where we developed a generalization of the Empirical Interpolation Method in order to couple data assimilation with reduced models. After giving a brief overview of the state of the art in this topic, I will summarize ongoing research in this direction related to the stability of the approximation and the inclusion of noise in the analysis of the scheme.

Adaptive wavelet methods for PDEs : the story, and some recent advances Valérie Perrier

Université de Grenoble

The concept of "Adaptive wavelet methods for PDEs" was born 26 years ago, in a seminal note of Yvon Maday and collaborators : "Adaptativité Dynamique sur bases d'ondelettes pour l'approximation d'EDPs" (C.R. Acad. Sci, 1991). For the first time the adaptivity was done a priori and not -as in other current methods- a posteriori: the time evolution of the wavelet coefficients of the solution was inferred from the knowledge of coefficients at previous times. From there, the wavelet adaptivity concept was a great success and was at the beginning of numerous and various numerical methods for solving PDEs. After a short overview on this topic, the talk will focus on more recent advances in the wavelet field, and particularly the use of divergence free wavelets for Optimal Transport.

Mathematical electroporation modeling, from cell scale towards clinical applications Clair Poignard

INRIA & Université de Bordeaux

Electropermeabilization (also called electroporation) is a significant increase in the electrical conductance and permeability of cell membrane that occurs when pulses of large amplitude (a few hundred volts per centimeter) are applied to the cells: due to the electric field, the cell membrane is permeabilized, and then nonpermeant molecules can easily enter the cell cytoplasm by transport (active and passive) through the electropermeabilized membranes. This phenomenon raises interesting questions in mathematical modeling, at different levels. I will present the main results obtained these last years in cell electropermeabilization, and how counter-intuitive observations can be explained by our model. I will also present the mathematical and numerical challenges to be tackled regarding the use of electroporation as anticancerous therapy.

From oil to glass, via a PhD with Yvon Maday

Nicole Poussineau

Saint-Gobain Recherche

I will explain why I have decided to make a PhD with Yvon Maday, what I have learnt and what is still useful now, 10 years later, working in Saint-Gobain research center. I will speak a lot about industrial applications, a little less about careers in industry, and little about academic collaborations.

Reduced Order Methods for parametric Fluid-Structure Interaction problems: applications to haemodynamics

Gianluigi Rozza

SISSA, Mathematics Area, mathLab International School for Advanced Studies Trieste, Italy We introduce the state of the art for reduced order methods in fluid-structure interaction parametric problems with monolithic and segregated approaches. Several aspects are taken into consideration: stability of the approximation, algebraic stability, reduced basis enrichment, fluid-structure interface management, reduced order coupling conditions between pressure of the fluid and stresses of the structure, as well as domain decomposition and an optimal flow control approach. The reduced order methods are based on POD-Galerkin approaches. An important feature of the problems is the parametric approach, as well as the capability to solve complex multiphysics coupled problems within an offline-online computational setting with limited costs.

Efficient and accurate spectral methods for PDEs with singular solutions **Jie Shen**

Purdue University and Xiamen University

The usual spectral methods will provide high-order accuracy for problems with smooth solutions. However, they may not work well for problems with singular solutions due to various facts such as corner singularities, non-matching boundary conditions, non-smooth coefficients. If the form of the singular expansion for the solution is known, we develop a Muntz Galerkin method which is based on specially tuned Muntz polynomials to deal with the singular behaviors of the underlying problems, and show that it provide optimal error estimates. On the other hand, if the Muntz Galerkin method is not applicable or efficient, we present a new extended spectral-Galerkin method which allows us to split it into two separate problems: one is to find an approximation for the smooth part by a usual spectral method, the other is to determine an approximation to the singular part with k terms by solving a $k \times k$ system. So the new method is very easy to implement, very efficient and is capable of providing very accurate approximations for a class of singular problems.

We will present ample numerical results for a variety of problems with singular solutions, including fractional PDEs, to demonstrate the effectiveness of our approaches.

On Adaptive Spectral Galerkin Methods with Dynamic Marking Marco Verani

MOX-Politecnico di Milano

The convergence and optimality theory of adaptive Galerkin methods is almost exclusively based on the Dorfler marking. This entails a fixed parameter and leads to a contraction constant bounded below away from zero. For spectral Galerkin methods this is a severe limitation which affects performance. We present a dynamic marking strategy that allows for a super-linear relation between consecutive discretization errors, and show exponential convergence with linear computational complexity whenever the solution belongs to a Gevrey approximation class. This is a joint work with Claudio Canuto, Ricardo H. Nochetto and Rob Stevenson.
